



Identification and Exclusion Multiple Outliers in GNC Microsystem

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Abstract: Monitoring the positioning reliability is of great importance for modern Global Navigation Satellite System (GNSS), especially for receivers which are on duty in multi-constellation Guidance Navigation and Control (GNC). Integrity and relevant techniques are gaining popularity among which many researches focus on receiver autonomous integrity monitoring (RAIM) referring to independent integrity without any external assistance. Therefore, the performances of RAIM have to be lucubrated in order to assure the positioning quality. An innovative method has been proposed for failure identification and exclusion in GNSS among GNC microsystem. This method is established on the evaluation of subsets precheck and residual vector adjustment to conduct RAIM. It is applicable to multi-constellation receivers carrying on FDE. Concerning on abundant visible satellites, we consider Geometric Dilution Precision (GDOP) for optimized satellite spatial geometry and manage to avoid matrix inverse operation by Newton efficient equivalence solution, which saves large quantity computational complexity in GNC microsystem. The main purpose is to improve the ability of GNSS receiver accuracy, availability, continuity and integrity. The multiple outlier hypothesis has also been evaluated to ensure the integrity by the statistical global and local tests of least squares residual. Simulation consequences imply that we are able to obtain fairly good behaviors when processing single and two outliers.

Keywords: GNC, GNSS, GDOP, RAIM, FDE

1. Introduction

With the development of chip design, advanced material, microfabrication and integrated packaging, microsystem technology achieves the advantages of intelligence, low cost, miniaturization, integration, mass production, clustering etc [1]. Generally speaking, the scope of microsystem includes roominess technologies, i.e. microelectronics, Micro-electromechanical Systems (MEMS) and microbial fluid science. Microsystem involves a fairly comprehensive discipline including micro machining, micro actuators, multisource sensor, signal processing, information decoding, and execution circuit, communication, power processing and System in Package (SIP) integration. The application of microsystem could be extended to industrial and military application. In this paper, the multisource sensors in Guidance Navigation and Control (GNC) microsystem mainly concentrate on the Global Navigation Satellite

System (GNSS). Fault tolerance could be achieved by appropriate redundancy algorithm through failure detection, exclusion and reconfiguration mechanisms in GNC microsystem.

Mainly four satellite navigation systems exist in the world, i.e. GPS of America, GLONASS of Russia, BDS of China and Galileo of Europe. GNSS industry has been revolutionized over the past two decades. Significant breakthroughs of positioning, velocity and timing (PVT) have been demanded and realized by many military industrial enterprises and scientific research institutions. Although achieving remarkable improves, industry demands still keep away from satisfied due to the limitation of single navigation system. Fortunately, more and more satellites will be on duty in space with the development of Galileo by the European Commission (EC) and European Space Agency (ESA). Meantime, as the Asia Pacific region has been covered by BDS in 2012, the new third generation BDS satellites will be launched one after the other in 2017~2022. With the

extensive application and popularization of GPS/BDS/Galileo, about 90 satellites will be arranged in the space and about 40 satellites will be tracked by the multi-constellation GNSS receiver channels at each positioning epoch [2]. With the complete co-operation of all these GNSS, greater levels of satellite accuracy, availability, continuity will be achieved and therefore integrity can be ensured significantly. However, if all the tracked satellites take part in normal working, it will give great pressure to the processing units significantly, especially in GNC microsystem. Thus, limited and optimal satellite spatial geometry distribution of Geometric Dilution of Precision (GDOP) should be introduced with high-efficiency. Moreover, we could not neglect the satellites with existing outliers at the same time. In order to eliminate faulted satellites, we generally take Receiver Autonomous Integrity Monitoring (RAIM) into consideration to make trouble removed and we cannot neglect simultaneous multiple outliers any more [3]. The main purpose is to ensure and promote the accuracy, availability, continuity and integrity in GNC microsystem.

In this paper, an innovative method has been proposed for failure identification and exclusion in GNC microsystem. This approach is established on the direct evaluation of subsets precheck and residual vector adjustment to conduct RAIM. While subsets precheck is carried out by fast satellite selection algorithm which reduces the calculation complexity by avoiding traditional matrix inverse operation. The reduction of calculation complexity plays an important role on GNC microsystem which extremely controls the power dissipation. In addition, subsequent residual vector adjustment is conducted to implement global and local statistical integrity testing. Unlike traditional single outlier processing methods, we could deal with multiple outliers simultaneously by proposed method. Next section, review of related algorithms would be discussed. Detailed description on the proposed fast satellite selection algorithm is introduced afterward. After that, RAIM and False Detection and Exclusion (FDE) is proposed. Analysis are then given in detail. The last section summarizes the full text with remarks on further research.

2. Literature Review

With the development of GNSS in GPS/BDS/Galileo, more than 40 satellites will be appeared at each positioning epoch for GNC receivers. Therefore, satellite selection plays an important role on decreasing calculation complexity for receivers with limited data processing units and enhancing positioning performance by optimal satellite geometric distribution in GNC microsystem. Satellite selection, which means selecting several certain satellites from all visible ones, plays a crucial role on improving positioning precision and reduces receiver computational complexity, especially for those GNC microsystem with limited processing units. Satellite selection could achieve the decrement of measurements, which can shorten the computing time [4]. Recent studies mainly focus on the calculation of GDOP.

Traditional GDOP indicates matrix direct inversion,

accompanied by large computational complexity. Yarlagadda made clear GDOP metric with its known bounds by using formal linear algebraic framework [5]. An approaching formula for fast computation of GDOP was produced by Zhu, which was applied to for four satellites merely [6]. It showed an urgent requirement to explore alternative methods except Zhu's algorithm. The reason lied in that most GNC receivers today could be able to track satellite signals fairly more than four satellites. The method of satellite selection by using Generalized Regression Neural Network (GRNN) was proposed [7]. In addition, statistics and machine learning methods, such as Support Vector Machine (SVM), Pace Regression (PR), Artificial Neural Network (ANN) and Genetic Programming (GP), were adopted to process the approximation calculation of GDOP and indicated that SVM and GP could achieve better performance than other algorithms [8]. However, Doong indicated that ANN and GM algorithm required larger quantity of training sequence and were likely to sample depletion. In this paper, a new fast satellite selection algorithm was created with the help of convex geometry theory under Gaussian distribution and uncorrelated hypothesis. The hypothesis of different measurement bias had been considered meantime [9]. Thinking over the time-consuming and power-saving calculation in GDOP, Teng proposed a closed-form calculation for multi-constellations, which could be adapted for the multi-GNSS constellations including two, three or four single ones [10]. Typical complex electromagnetism environments in urban areas and their effects on fast satellite selection algorithms were heated discussion in [11]. Moreover, with the development of modern GNSS plans, the influence of the multi-constellation had been analyzed theoretically [12]. The influence factors of GDOP calculation by adopting different satellite selection algorithms were analyzed and summing-up that the value of GDOP will be reduced with the number of satellites for PVT increases. In order to decrease the large number calculation of matrix inversion and taking advantage of Newton's identities by the theory of symmetric polynomials, a simple closed-form formula for GDOP efficient calculation was proposed and could obtain fairly the same precision compared with matrix direct inversion [13], which enhances the PVT efficiency of fast satellite selection significantly in GNC microsystem.

In order to achieve the target of reliability and precision in PNT of GNC microsystem, we take RAIM/FDE into consideration simultaneously. Integrity monitoring in GNC could be composed of evaluating the residuals between the pseudorange and geometric distance from GNC receiver to satellites under each positioning epoch, aiming at detecting and excluding probable outliers. Traditional RAIM algorithm mainly focused on processing with single outlier, far from satisfying the requirement of modern GNSS receivers [14]. The FDE in GNC receiver was often accompanied with RAIM. The function of FDE required to provide its current protection capability which was generally known as the horizontal protection level (HPL). Moreover, the FDE function also need an estimation of current horizontal position uncertainty, namely

measurement inconsistency, i.e., horizontal uncertainty level (HUL). Before conducting FDE, we also need conducting reliability monitoring aiming at detecting outliers and estimating the effects that the undetected outlier may influence on the exist normal ones.

We manage to deal with multi-outliers in GNC microsystem on each positioning epoch. We also take fast satellite selection algorithm and RAIM into consideration for GNC integrity enhancement, mainly achieving by subsets precheck and residual vector matrix adjustment to accomplish above targets. The availability of RAIM is also conducted in case the influence of geometric distribution. Afterwards, global and local testing would be achieved after fast satellite selection for FDE. In summary, we can obtain fairly good behaviors with the help of above advantages.

The novelty is that we could improve positioning accuracy and lower receiver computational complexity by fast satellite selection in GNC microsystem, relying on both Newton's identities for GDOP and outlier efficiency FDE. More available and reliable navigation algorithm will be achieved in RAIM, resulting in global and local monitoring in conjunction with fast satellite selection algorithm.

3. Fast Satellite Selection in GNC Microsystem

3.1. Analysis of GNSS Positioning

GNSS receivers conduct the user position from iterative bias measurements of satellite-to-user ranges. Pseudorange measurement transferred by satellite navigation message is shown as follows:

$$\rho^{(n)} = r^{(n)} + \delta t_u - \delta t_s^{(n)} + I_{ono}^{(n)} + T_{rono}^{(n)} + \varepsilon_\rho^{(n)} \quad (1)$$

where $r^{(n)}$ represents the geometric distance between the receiver and n th visible satellite (GPS, BDS or Galileo), δt_u represents receiver clock bias, $\delta t_s^{(n)}$ represents satellite clock bias, $I_{ono}^{(n)}$ stands for ionospheric delay and $T_{rono}^{(n)}$ stands for tropospheric delay, $\varepsilon_\rho^{(n)}$ stands for other indirect measurement errors, e.g., group delay, antenna phase noise, multipath effect, receiver native thermal noise etc. The time factor is carried out by light speed and the unit of all above factors is meter. Aiming at constructing receiver positioning equations, the revised pseudorange measurement could be modified as follows:

$$\rho_c^{(n)} = \rho^{(n)} + \delta t_s^{(n)} - I_{ono}^{(n)} - T_{ono}^{(n)} \quad (2)$$

$$r^{(n)} = \|X^{(n)} - X\| = \sqrt{(x^{(n)} - x)^2 + (y^{(n)} - y)^2 + (z^{(n)} - z)^2} \quad (3)$$

where $X = [x, y, z]^T$ represents receiver coordinate in ECEF, $X^{(n)} = [x^{(n)}, y^{(n)}, z^{(n)}]^T$ stands for satellite coordinate vector, $r^{(n)}$ represents the baseline distance between the tracked satellite and receiver. Modified pseudorange measurement $\Delta\rho$ is inferred as follows:

$$\Delta\rho = \rho_c^{(n)} - r^{(n)} - \delta t_u \quad (4)$$

The corrected pseudorange measurements are linearly optimized by Taylor first order expansion on assumption that the linearization point is sufficiently close to the true receiver position.

$$\Delta\rho = H \cdot \Delta X + \varepsilon \quad (5)$$

where H stands for the line of sight (LOS) matrix and ε stands for measurement noise vector matrix presuming to be Gauss normal distribution $\varepsilon \sim N(0, \Sigma)$. Therefore, the best linear unbiased estimation (BLUE) would be the most approximate estimation for actual implementation due to that it could yield the lowest estimation error among all linear estimation ones. On assuming that $H^T \Sigma^{-1} H$ is nonsingular matrix, the BLUE of ΔX is shown as follows:

$$\Delta X = (H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1} \Delta\rho \quad (6)$$

And the positioning results of receiver is carried out iteratively until approximating predefined residual threshold and it will obtain convergence. The positioning is carried out by incrementing ΔX to previous estimation coordinates:

$$X = X_0 + \Delta X \quad (7)$$

Usually speaking, we regard GDOP as the measurement error magnification from the pseudorange to receiver positioning and the smaller GDOP, the better positioning outcomes. Traditional GDOP calculation is shown as follows:

$$GDOP = \sqrt{\text{trace}\left(\left(H^T H\right)^{-1}\right)} = \sqrt{\frac{\text{trace}\left(\text{adj}\left(H^T H\right)\right)}{\det\left(H^T H\right)}} \quad (8)$$

where $\text{trace}()$ represents matrix trace computing, $\text{adj}()$ stands for adjoint matrix estimation and $\det()$ stands for the determinant operation. The calculation of GDOP is a powerful tool for positioning of GNSS receiver. All receivers take full advantage of advanced algorithms to choose the best group of satellites in view. Meantime, positioning accuracy would be evaluated by the criterion of GDOP. In this paper, we put forward a new fast satellite selection algorithm for multi-constellation with the help of both Newton's identities for GDOP fast computation and optimal geometric distribution for receiver positioning less cycle calculation.

3.2. Approximate Formula for GDOP Fast Computation

Yarlagadda et al. analyzed the boundary of GDOP by theoretical mathematics [错误!未定义书签。]. When four satellites are adopted to achieve iterative process of receiver positioning, we could achieve that:

$$GDOP \geq \sqrt{2} \quad (9)$$

We come onward by denoting the four eigenvalues of $H^T H$ by $\lambda_i, i = 1, 2, 3, 4$. Thus,

$$GDOP = \sqrt{\text{trace}(H^T H)} = \sqrt{\sum_{i=1}^4 \frac{1}{\lambda_i}} \geq 2 \sqrt{\left[\prod_{i=1}^4 \frac{1}{\lambda_i} \right]^{1/4}} = \frac{2}{|H^T H|^{1/8}} \quad (10)$$

The value of GDOP would reduce with the number of tracked satellites increases in multi-constellation GNSS. Zhu et al. raised a closed-form formula for GDOP fast computation. Considering $E_{i,j} = e_{i1}e_{j1} + e_{i2}e_{j2} + e_{i3}e_{j3} + 1, 1 \leq i < j \leq 4$ where $e_{i1}e_{j1}, e_{i2}e_{j2}, e_{i3}e_{j3}$ stands for the direction vector in matrix H , meeting that $e_{i1}^2 + e_{i2}^2 + e_{i3}^2 = 1$. Suffering a series of simplification, finally we could achieve following formula:

$$GDOP = \sqrt{\frac{16+b+c}{a+b+2c}} \quad (11)$$

where a, b, c stands for intermediate factors shown as follows:

$$\begin{aligned} a &= (E_{12}E_{34} + E_{13}E_{24} - E_{14}E_{23})^2 - 4(E_{12}E_{34}E_{13}E_{24}) \\ b &= 16 - 4(E_{12}^2 + E_{13}^2 + E_{14}^2 + E_{23}^2 + E_{24}^2 + E_{34}^2) \\ c &= 2[E_{12}(E_{13}E_{23} + E_{14}E_{24}) + E_{34}(E_{13}E_{14} + E_{23}E_{24})] \end{aligned} \quad (12)$$

Starting from elements of H , it has been proved that 39 multiplications, 34 additions, 1 division and 1 square root are needed for GDOP computation in Eq. 11. And this is less than half of the direct GDOP computation in Eq. 8. However, it just meets the case of 4 satellites due to critical equality available only for this dimension. Simon et al. proposed a closed-form formula for estimating GDOP by ANN [15]. Assuming that $M = H^T H, M \in R^{4 \times 4}$ and it is a symmetrical matrix. We could obtain four characteristic factors $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. Since M is a non-singular matrix and could be expressed as follows:

$$GDOP = \sqrt{\text{trace}(M^{-1})} = \sqrt{\lambda_1^{-1} + \lambda_2^{-1} + \lambda_3^{-1} + \lambda_4^{-1}} \quad (13)$$

On purpose of estimating GDOP in Eq. 13, a special transformation of features is conducted as follows:

$$\begin{aligned} h_1(\lambda) &\equiv \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = \text{trace}(H) \\ h_2(\lambda) &\equiv \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 = \text{trace}(H^2) \\ h_3(\lambda) &\equiv \lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \lambda_4^3 = \text{trace}(H^3) \\ h_4(\lambda) &\equiv \lambda_1 \lambda_2 \lambda_3 \lambda_4 = \det(H) \end{aligned} \quad (14)$$

where h_1, h_2, h_3 stands for the first, second and third degree of the eigenvalues, h_4 represents the determinant of matrix M . Eq. 13 suffers lots of evolution, such as ANN, GP etc. Doong carried out an algebraic polynomial for GDOP fast calculation [错误!未定义书签。].

$$GDOP = \sqrt{\frac{\lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}} \quad (15)$$

The symmetric polynomial stands for the sum of all k th degrees of the eigenvalues, we assume following formula:

$$p_k(X_1, X_2, \dots, X_n) = X_1^k + X_2^k + \dots + X_n^k \quad (16)$$

Generally, the symmetric polynomial $p(X_1, X_2) = X_1^2 X_2 + X_1 X_2^2 + X_1 X_2$ could be achieved by follows:

$$p(X_1, X_2) = \frac{1}{2} p_1^3 - \frac{1}{2} p_1 p_2 + \frac{1}{2} p_1^2 - \frac{1}{2} p_2 \quad (17)$$

where p_k is defined in Eq. 16. And the third degree elementary in Eq. 14 could be written as follows:

$$e(X_1, X_2, X_3, X_4) = X_1 X_2 X_3 + X_1 X_2 X_4 + X_1 X_3 X_4 + X_2 X_3 X_4 \quad (18)$$

Using Newton's identities from the theory of symmetric polynomial, it indicates that

$$e = \frac{1}{3} \left[\frac{1}{2} (p_1^2 - p_2) p_1 - p_1 p_2 + p_3 \right] \quad (19)$$

Thus, a closed formula for GDOP calculation regard as universality is given by:

$$GDOP = \sqrt{\frac{0.5h_1^3 - 1.5h_1 h_2 + h_3}{3h_4}} \quad (20)$$

4. Integrity Monitoring

4.1. Model Analysis

Integrity monitoring indicates the ability to warning the receiver timely when the navigation system error exceeds the normal threshold. In order to treat the system as a robust integrity monitoring carrier, we continue to improve the integrity monitoring algorithms and functions. In this paper, integrity monitoring is carried out by both RAIM and FDE, including global test for detecting inconsistencies and local test for localizing and ultimately mitigating errors recursively. In addition, we combine fast satellite selection algorithm with RAIM/FDE simultaneously, which not only lowers the computational complexity but also enhances the precisions with reliable positioning results.

Achieving a valid measurement solution, observational residuals are defined by the difference between the estimated measurements, combing of which indicates the attitude to agree with each other. Thus, the residual is an useful tool for evaluating the quality among the estimated parameters. When obtaining the redundant observations, we could work out residuals of the pseudorange by Weighted Least Squares (WLS). The residual vector \hat{b} is shown as follows:

$$\begin{aligned} \hat{\mathbf{b}} &= \Delta\rho - H \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\delta t_u \end{bmatrix} \\ &= \Delta\rho - H(H^T\Sigma^{-1}H)^{-1}H^T\Sigma^{-1}\Delta\rho \\ &= S(\Delta\rho + \boldsymbol{\varepsilon}) \end{aligned} \quad (21)$$

where $S = I - H(H^T\Sigma^{-1}H)^{-1}H^T\Sigma^{-1}$ and $\boldsymbol{\varepsilon}$ stands for the measurement noise vector. At the same time, we define Weighted Sum of Squares for Error (WSSE) as follows:

$$\boldsymbol{\varepsilon}_{WSSE} = \hat{\mathbf{b}}^T \Sigma^{-1} \hat{\mathbf{b}} \quad (22)$$

where $\boldsymbol{\varepsilon}_{WSSE}$ is the length square of weighted residual vectors. In fact, $\Delta\rho$ is a zero mean vector, we can achieve the least value of $\boldsymbol{\varepsilon}_{WSSE}$ from weighted residual vector which is treated as the various degree of consistency between different measurement observations and obeys chi-square distribution with $N - 3 - m$ Degree of Freedom (DOF) where N is the number of satellites for receiver positioning and m stands for the number of GNSS constellation. Otherwise, $\boldsymbol{\varepsilon}_{WSSE}$ is a noncentrality chi-square distribution random vector distributed with the noncentrality parameter of $\lambda = \sqrt{\hat{\mathbf{b}}^T \Sigma^{-1} S \hat{\mathbf{b}}}$. We define the false alert rate P_{FA} and missed detection rate P_{MD} as follows:

$$\begin{cases} P_{MD} = P(\text{teststatistic} < T_{thr} / H_1) \\ P_{FA} = P(\text{teststatistic} \geq T_{thr} / H_0) \end{cases} \quad (23)$$

where T_{thr} represents the detection threshold and is resolved by P_{FA} and DOF. H indicates event assumption and H_1 represents satellite system including outliers while H_0 represents regular solution. With the probability distribution function of chi-square and non-centrality chi-square distribution, P_{FA} and P_{MD} are shown as follows:

$$P_{FA} = \int_{Thr}^{\infty} \frac{x^{\frac{(N-4)}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{N-4}{2}} \Gamma(\frac{N-4}{2})} dx \quad (24)$$

$$P_{MD} = \int_0^{Thr} \frac{1}{2} e^{-\frac{x+\lambda}{2}} \left(\frac{x}{\lambda}\right)^{\frac{(N-4)}{2}-1} I_{\frac{N-4-2}{2}}(\sqrt{\lambda x}) dx \quad (25)$$

where $\Gamma(\alpha)$ stands for Gamma function, and

$$I_a(y) = \left(\frac{y}{2}\right)^2 \sum_{j=0}^{\infty} \frac{(y^2/4)^j}{j! \Gamma(a+j+1)}$$

represents the first order corrected Bessel function.

The relationship between P_{FA} and P_{MD} is shown in Figure 1. The scope of lilac represents P_{MD} and green section represents P_{FA} . A contradiction occurs between P_{FA} and P_{MD} . The threshold is often determined by P_{FA} and DOF. Consequently, any increasement of the non-central parameters λ will contribute to lower the missed detection probability.

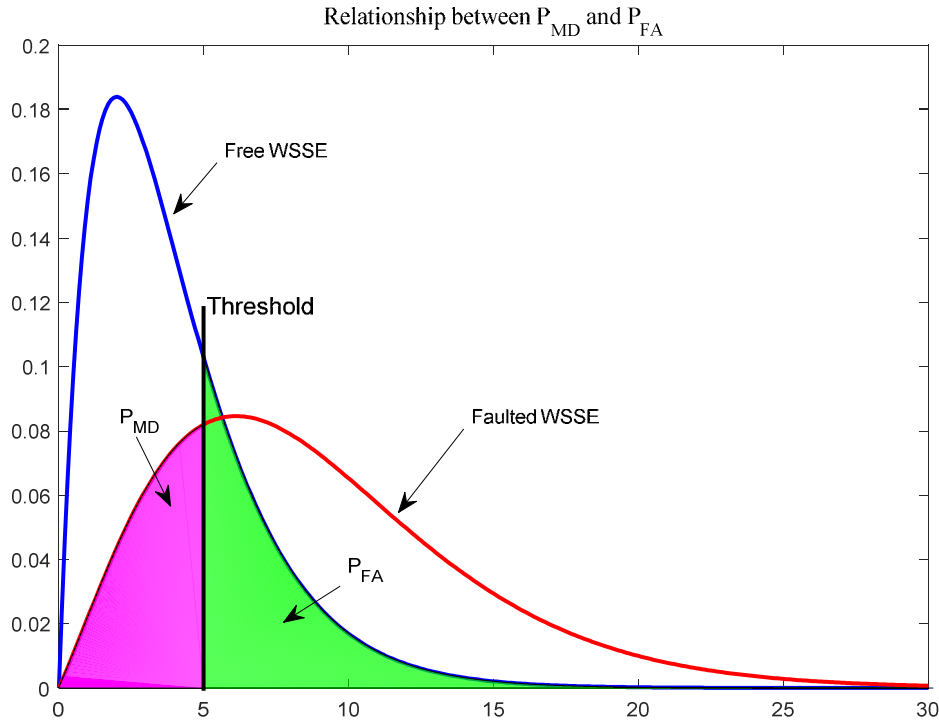


Figure 1. Relationship between P_{FA} and P_{MD} .

The perspective of probability density function (pdf) between GNSS integrity and outlier detection, protection level and integrity risk are considered in Figure 2.

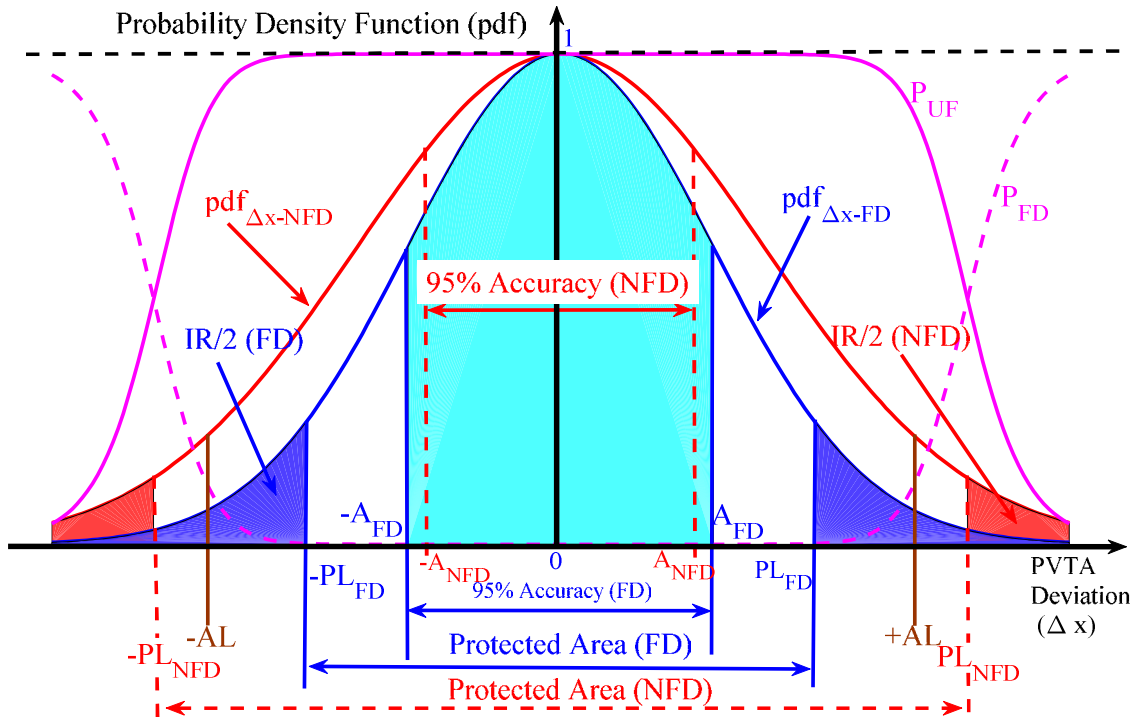


Figure 2. Relationship on integrity monitoring.

The x-axis Δx stands for the deviation of position, velocity, timing and attitude (PVTA). In order to carry out different application industry, we adopt different alert limit (AL), such as NPA, APV, CAT etc. The inferior magenta dashed curve represents P_{FD} , which stands for the detection ability of the navigation bias. Considering P_{FD} , the upper magenta dashed curve stands for the probability of undetected failure P_{UF} . P_{UF} is approximate 1 whose deviation is close to the zero and the most difficult to detect. The relationship between P_{FD} and P_{UF} is shown as follows.

$$P_{FD} + P_{UF} = 1 \tag{26}$$

The red solid line stands for the pdf of No Failure Detection (NFD). Usually, the Protection Level of NFD is larger than that of AL. After conducting RAIM, the pdf of Failure Detection (FD) is shown with the blue solid line. The PL of FD is less than that of AL. Both sides of the blue and red bell shape region stands for the integrity risk (IR) when the navigation results cannot be ensured by the pre-establish P_{MD} and P_{FA} . The protected area is the region less than PL including PL_{FD} and PL_{NFD} . From above figure, we could achieve that when we enhance the FD, the 95% accuracy A_{FD} is enhanced compared with NFD. Moreover, the risk of navigation continuity is enhanced at the same time, which indicates that the integrity is suffering risk of accuracy and continuity.

4.2. Protection Level Estimation

In order to carry out RAIM, we would also take the integrity availability into consideration. PL indicates an upper bound and require that the position error should not exceed it. PL implies an important factor to determine the availability of RAIM.

The performance of RAIM could only be considered in premise that the PL is less than the alert limit (AL) for a certain navigation task. The PL consists of satellite geometry distribution, the Minimum Detectable Error (MDE), derived from the probability of P_{MD} , the threshold for P_{FD} and statistical measurements. Current RAIM method mainly concentrates both the probability of P_{FA} and P_{MD} . The probability of P_{FA} is used to evaluate the threshold. The threshold with the P_{MD} is together used to determine the minimum detectable error P_{bias} , from which the Horizontal Protection Level (HPL) is determined by projecting P_{bias} from the measurement region to the position region using the geometry factor [16]

$$HPL = SLOPE_{max} \times P_{bias} \tag{27}$$

The horizontal position error (PE_H) is shown as follows:

$$PE_H = \sqrt{(A_{11}\epsilon_1 + \dots + A_{1n}\epsilon_n)^2 + (A_{21}\epsilon_1 + \dots + A_{2n}\epsilon_n)^2} \tag{28}$$

where $A = (H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1}$ and $A_{i,j}$ stands for the i th row and j th column element. The ratio of PE_H to the test statistic, usually referred as $SLOPE$, is regarded as the following equation:

$$SLOPE = \frac{PE_H}{WSSE} = \sqrt{\frac{(A_{11}^2 + A_{12}^2)\epsilon_1^2 + \dots + (A_{1n}^2 + A_{n2}^2)\epsilon_n^2}{\epsilon^T S \epsilon}} \tag{29}$$

The maximum slope $SLOPE_{max}$, could be evaluated as follows:

$$SLOPE_{max} = \text{Max}_i (SLOPE(i)) \tag{30}$$

We should realize that the satellite with the largest

slope $SLOPE_{max}$ is the most difficult to detect because it yields the smallest test statistic. The actual $SLOPE$ value contains noise from the lower $SLOPE$ satellites. Only when the HPL is less than that of HAL, we could achieve RAIM performance. Concerning the relationship between HPL and HUL, detailed explanation is introduced in [17]. The VPL is shown as follows:

$$VPL = x_v \pm k_v \cdot \sigma_v \quad (31)$$

where x_v represents the third element in East, North, Up (ENU). $k_v = 5.33$ for a 10^{-7} normal integrity risk and σ_v is a measurement of vertical accuracy derived from the covariance of the position estimate shown as follows [18].

$$\sigma_v^2 = [(H^T \Sigma^{-1} H)^{-1}]_{3,3} \quad (32)$$

Thus, the overall VPL will be chosen such that it defines an interval around the all-in-view estimated position including all the partial solution ranges.

5. Receiver Autonomous Integrity Monitoring

To achieve the objectives of detecting and excluding outliers, we try to carry out the combination of global and local test. We temporarily use ε_{WSSSE} as test statistics in WLS. In global test, the null-hypothesis H_0 represents the adjustment model that it is a normal solution and no outliers exist. Otherwise, we will carry out local test with more specific statistical tests for possible outlier detection and isolation. After FDE, we would conduct fast satellite selection again to ensure the number of satellites for position resolving and reliability depending on robust satellites.

5.1. Global Test

Global test for detecting inconsistent outliers is based on the statistics of ε_{WSSSE} which obeys central chi-square distribution with DOF of $N - p$ in assumption that the measurements are Gaussian normally distributed $\varepsilon_{WSSSE} \sim N(0, \Sigma)$, where N denotes the number of satellites for receiver positioning and p is the number of parameters to be estimated, usually valued 4, i.e., X, Y, Z coordinate and receiver clock bias. In this paper, we conduct GPS/BDS/Galileo due to the similar positioning mechanism and does not consider GLONASS. In addition, the proposed algorithm could also be implemented to the case of GLONASS by adequate modification, i.e., coordinate transformation between PZ-90 and WGS-84. However, among different navigation system of GPS, BDS and Galileo, we could achieve the same clock bias by tracked broadcast navigation message clock factor UTC_δ . If the statistic exceeds the predefined threshold $\chi_{1-\alpha, N-p}^2$, where α stands for the probability of P_{FA} , we would be in favor of H_1 and reject H_0 . In this situation, inconsistency satellites in the observations would be detected and we would carry out local test for possible outlier exclusion.

5.2. Local Test

The reason for the rejection of H_0 is the appearance of outlying observations. Strict residual vector testing is easy achieved under the assumption of only one outlier in the current epoch. And we can bring the local test into conduction iteratively until the statistics of global test does not exceed the threshold. A posteriori variance analyzes the consistency between the estimation and observations. The residual vector $\hat{\mathbf{b}}$ could be standardized as follows:

$$v_i = \frac{\hat{b}_i}{\sqrt{(C_{\hat{b}})_{ii}}}, \quad i = 1, 2, \dots, N \quad (33)$$

where \hat{b}_i represents the i th element of the residual vector and $(C_{\hat{b}})_{ii}$ stands for the i th diagonal element of the covariance matrix in residual $C_{\hat{b}}$. The covariance matrix of the residuals is evaluated as follows:

$$C_{\hat{b}} = \Sigma - H(H^T \Sigma^{-1} H)^{-1} H^T \quad (34)$$

After standardization operation, we could ignore v_i which indicates the largest residual measurement simultaneously. Then we could put another good satellite of low GDOP into effect by fast satellite selection algorithm and consider global test into effect again until there exist no test statistics exceeding the predefined threshold. The standardized residuals are then normally distributed with zero mean vector when $H_{0,i}$ is correct, and with a non-zero expectation otherwise. Thus, the local test is carried out based on the following comparison

$$v_i \leq n_{1-\frac{\alpha_0}{2}} \quad (35)$$

With rejection of $H_{0,i}$ if the critical value exceeds the threshold. When H_0 of the global test does not succeed, the local test is conducted for FDE, and thus, the observation with the largest value of v_i is focused and possibly rejected. Due to the correlation of the residuals, a cross error in an observation may spread over all the other residuals. If a blunder is large enough to cause many other reliability test failures, resulting in many alternatives, which is very important to conduct test statistics of residual vector in order to ensure the abandon of possible outliers. After troubleshooting of an outlier, the parameter estimation, global and local test, could be repeated for the positioning epoch until no more outliers could be identified.

5.3. False Detection and Exclusion

In order to conduct FDE, we should realize that if there are m outliers suspected in a navigation task, a redundancy of at least $m + 1$ is needed to possibly identify. In actual engineering application, an error along with large residual may influence other measurements significantly, erroneous rejection of a good observation is possible. Moreover, if more than one observation is rejected, the iterated RAIM process

should considerate earlier rejected ones again. The primary process could be summarized from global, local test until the final statistics of global test is less than the predefined threshold. Due to one abnormal outlier may have influence on another normal measurement, we should take the possible outliers into consideration again to prevent excluding normal observations. In order to ensure the precision of positioning, we should also consider the number of satellites for position resolving. We will put fast satellite selection algorithm again to repair rejected outliers. In this case, fast satellite selection algorithm will take full advantage of existing satellite information for PVT, and according to the characteristic of optimal satellite geometry distribution, we will select the

satellite which has similar elevation and azimuth of rejected one. Entirely, we would conduct global test again to ensure the reliability of observations.

The flowchart in Figure 3 indicates the detailed algorithm flow of RAIM and FDE procedure based on the global and local test. In addition, we effectively combine fast satellite selection algorithm with RAIM/FDE. The method takes advantage of global test to identify inconsistency residual vector and consider the local test to identify and exclude possible erroneous outliers. Both global and local test are then carried out recursively until no more erroneous measurements exist and the solution is indicated as reliable.

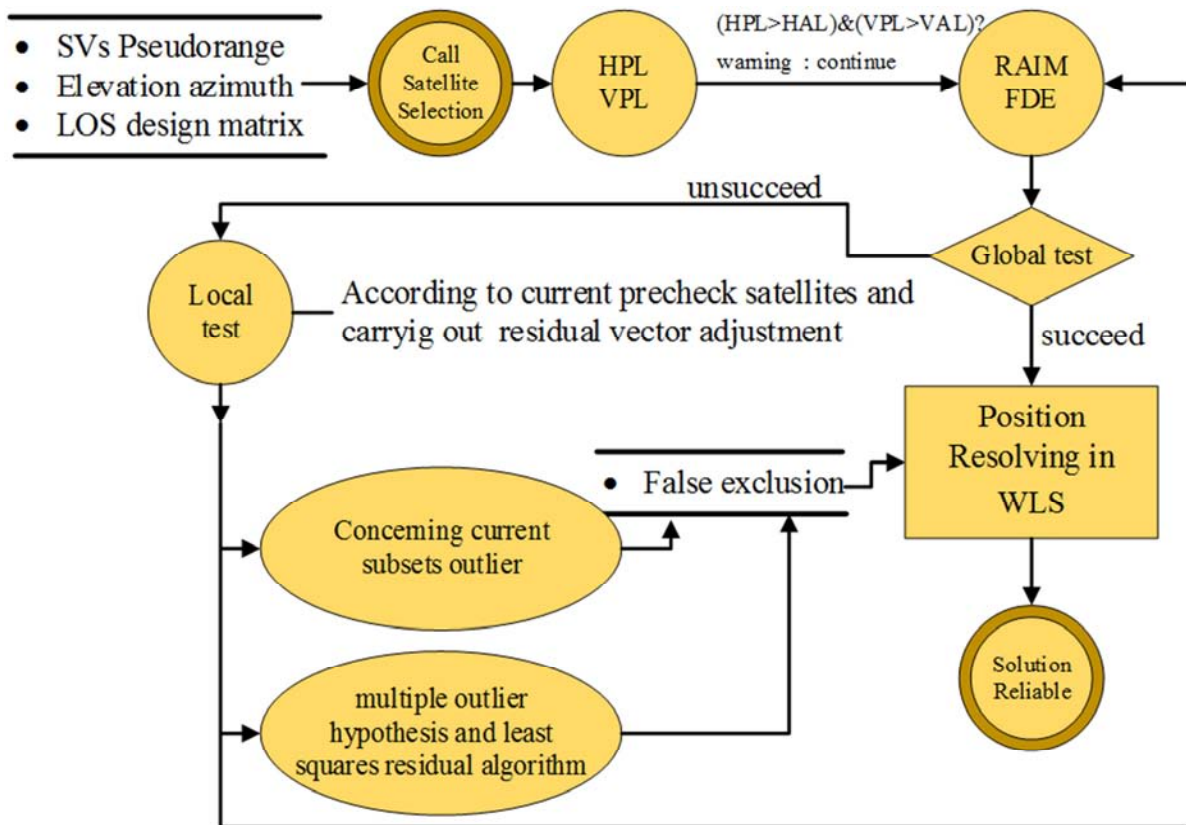


Figure 3. Algorithm flow combining fast satellite selection and RAIM/FDE in WLS.

Moreover, due to the outlier with large observation bias has an effect on other normal measurements, we would consider the reconsideration of earlier rejected ones. It is carried out by reconsidering all of the excluded measurements and performing global test to find the measurements which could be considered back into the PVT. Thus, a measurement which has been excluded in earlier local test is used again for positioning if the global test does not exceed the threshold upon its tentative inclusion. This is followed by fast satellite selection again to ensure the precision and robustness navigation following the principle that the more satellites for positioning, the more precision receiver could achieve. Due to the importance of the measurements to the geometry of solution, it is desirable to minimize the number of unnecessary exclusions. It is also

constructive to select the satellites with similar elevation and azimuth compared with rejected outliers according to the mechanism of optimal satellite spatial geometric distribution.

6. Simulation Performance and Analysis

6.1. Fast Satellite Selection Algorithm

In this section, actual GPS/BDS/Galileo navigation messages have been achieved from International GNSS Service (IGS) products. The simulation data is collected on 15th, Jan, 2020 from BCEmerge. The number of visible satellites (NVS) in global is shown in Figure 4. The mask angle is 5°.

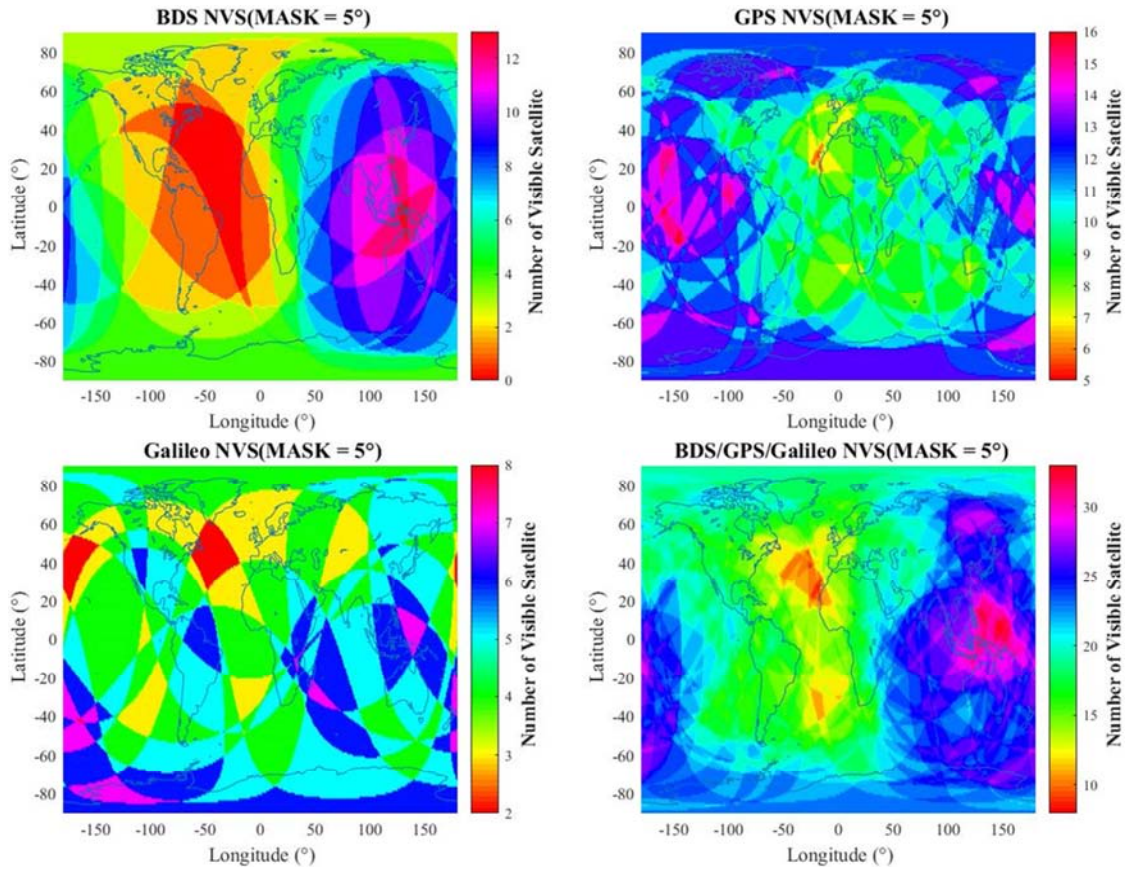


Figure 4. Number of visible satellites (NVS) in GPS/BDS/Galileo.

In actual engineering application, more than 20 satellites exist in three constellation of GPS/BDS/Galileo and could provide good integrity monitoring implementation. Although optimal satellite selection algorithm could obtain the best results while accomplishing the largest computational

complexity meantime. The detailed introduction of Quasi-Optimal satellite selection algorithm is in [19]. The performance of different satellite selection algorithm is shown in Figure 5.

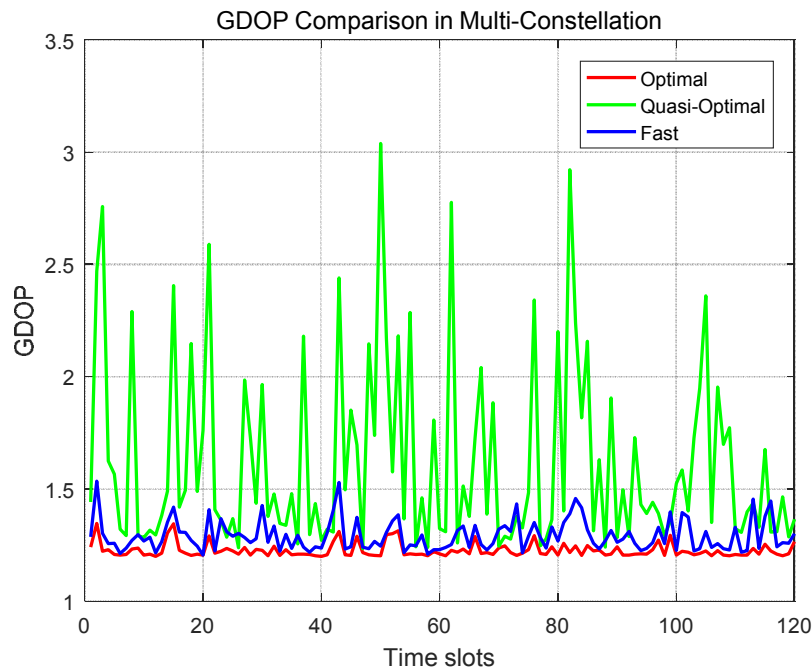


Figure 5. GDOP between Optimal, Quasi-optimal and Fast satellite selection algorithm.

The numerical value of GDOP indicates the smallest when adopting optimal satellite selection, while it includes the largest computational complexity at the same time. In Figure 5, it indicates that the value of GDOP by fast satellite selection algorithm is 0.1 larger than that of optimal results on average and 0.5 less than that of quasi-optimal satellite selection algorithm. Assuming that the pseudorange measurement deviation in dual frequency receiver is 2m, then the transformed positioning errors of fast satellite selection algorithm is 0.2m, nearly no influence on receiver PVTs compared with optimal satellite selection algorithm, while it could save large quantity computational complexity.

6.2. Integrity Monitoring—RAIM/FDE

Integrity enhancement depends on the direct evaluation of subsets precheck by above fast satellite selection and residual vector adjustment for RAIM and FDE in order to detect and exclude outliers. Global and local test is put into effect when detecting possible outliers by least squares residual. In this section, we add pseudorange errors artificially and slowly with one and two outliers. The receiver motion trajectory is shown in Figure 6.

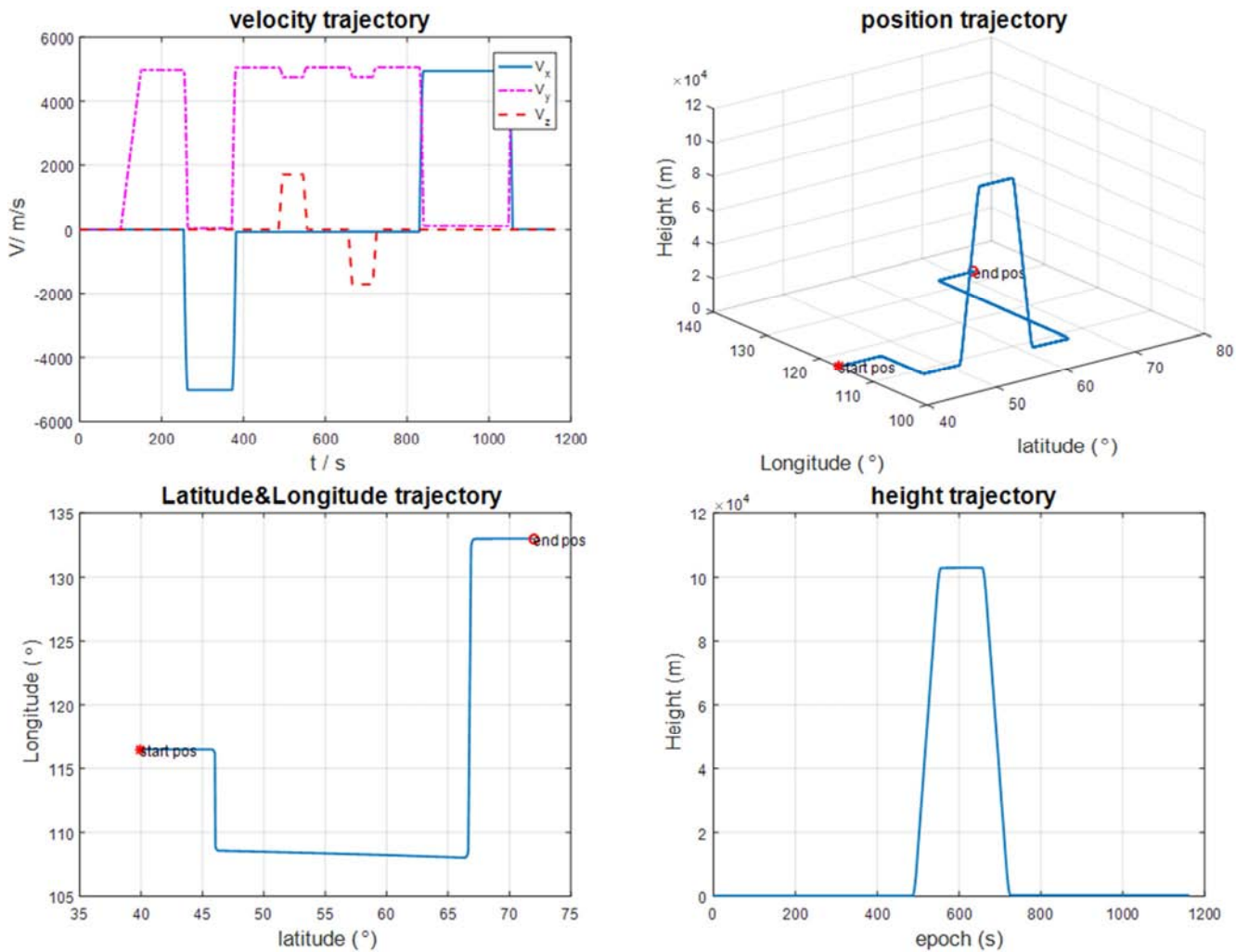


Figure 6. Receiver motion trajectory with position and velocity.

The start position is set in Beijing, China. The movement state is divided into following aspects:

- (a) keep static 100s (b) maintain accelerate state 100s with $a=5g$ ($g=9.78m/s^2$) (c) turn right with 90° (d) keep uniform state 100s (e) turn left with 90° (f) maintain climb state with 20° (g) keep uniform state 100s (h) descent state with 20° (i) turn right with 90° (j) keep uniform state 200s (k) turn left with 90° (l) keep uniform state 100s.

Upon proposed simulation scenes, we add pseudorange bias (PR) to the tracked satellites randomly and the outlier is

set one and two respectively. We also take the ionospheric delay, tropospheric delay, satellite clock bias, receiver noise etc into consideration. The normal positioning accuracy is compared with predefined simulation trajectory. The statistical results without outliers is shown in Figure 7. Due to that least squares residual vector algorithm is very sensitive when the PR is larger than 10m. Thus, we add PR bias=10m with one outlier and two outliers randomly. The position results assisted by RAIM/FDE is shown in Figure 8.

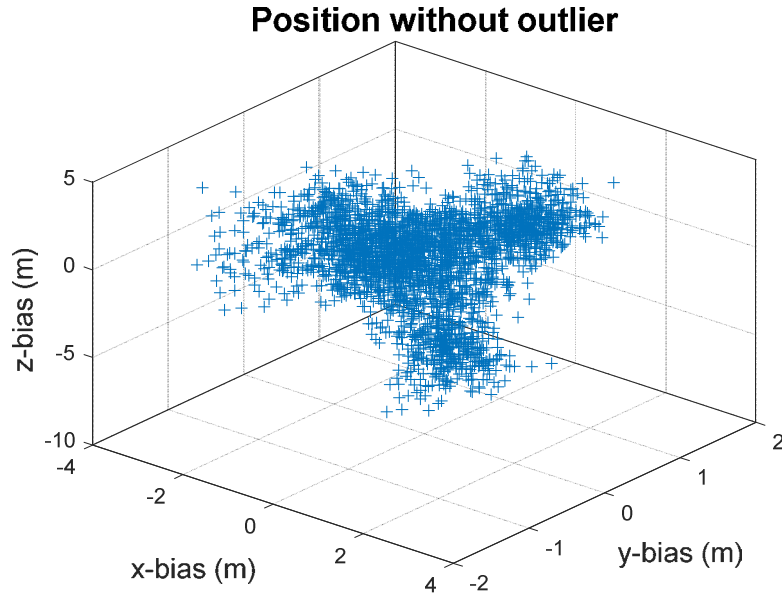


Figure 7. Normal PVT results.

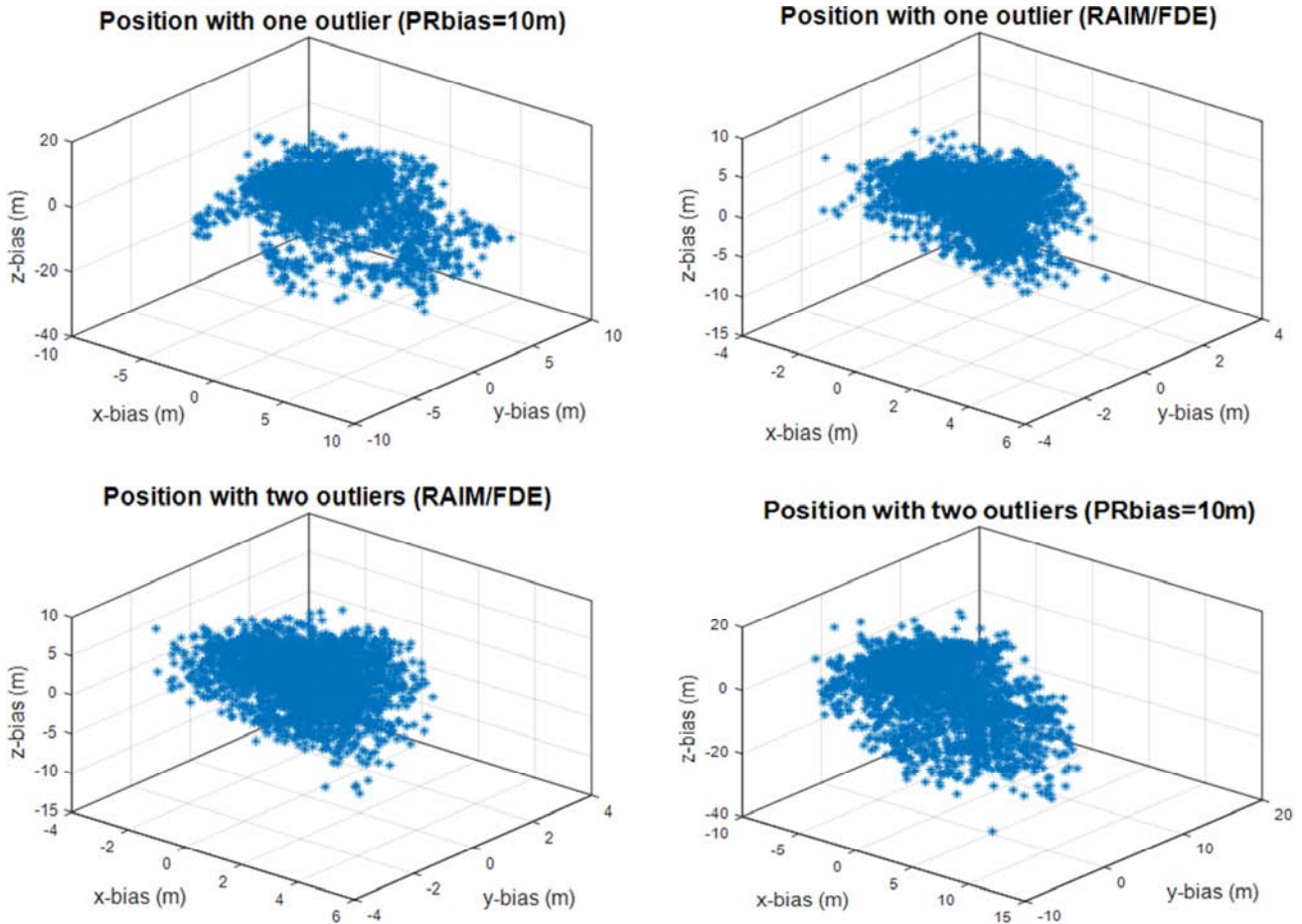


Figure 8. Results compared with outlier and RAIM/FDE.

When there are no outliers, the receiver X/Y/Z position bias in ECEF compared with predefined trajectory is 0.93114m, 0.60822m and 1.4082m respectively. After interfered by pseudorange observation bias with one outlier, the X/Y/Z is 2.1557m, 1.6953m and 4.1427m. Direct consequence is achieved by subsets precheck and residual vector adjustment

in RAIM/FDE and the X/Y/Z is 1.0163m, 0.69228m and 1.6843m, nearly the same with original outcomes with noise model consideration. Similarly, the X/Y/Z is 2.8689m, 2.4039m and 6.5082m with two outliers. After conducting RAIM/FDE, the X/Y/Z is 1.0696m, 0.77303m and 1.923m respectively. The VPL results in global region is shown in

Figure 9. One and two outliers are random added in the observation measurements. The integrity risk is set 10^{-7} and the mask is 5° . The maximum VPL is 39.6264m, the minimum VPL is 17.9761m and the mean VPL is 26.1008m. Considering fast satellite selection with subsets precheck and

global and local testing for outlier exclusion, the maximum, minimum, mean VPL is 34.2495m, 12.5992m, 20.724m respectively, which nearly enhance 20.6% mean VPL value in global region.

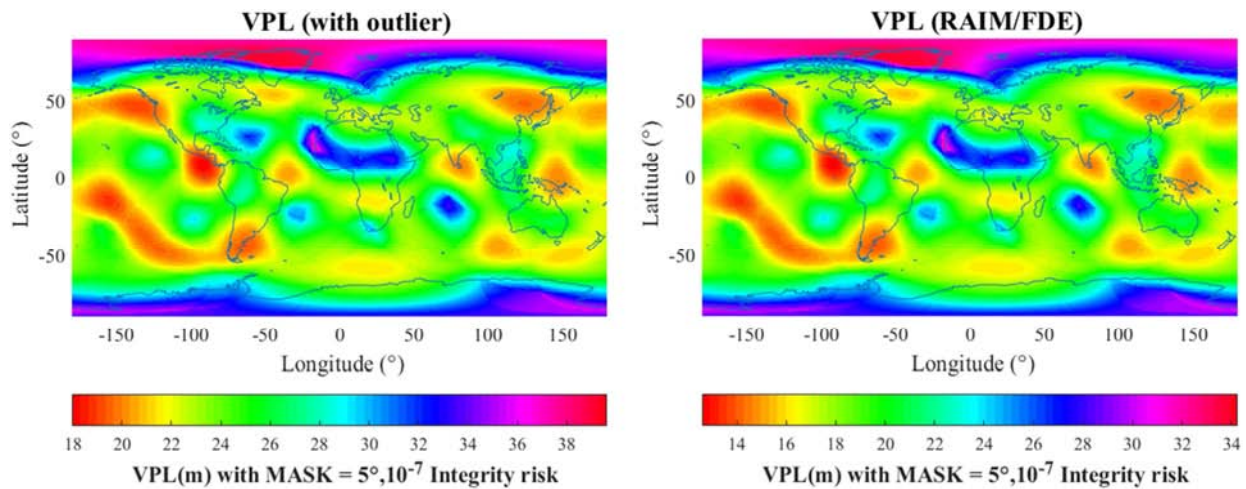


Figure 9. VPL results comparison in global region.

In summary, fast satellite selection algorithm is compared with traditional optimal and quasi-optimal satellite selection algorithm. Static and dynamic motion trajectory is forward and back contrasted by the subsets precheck and residual vector adjustment. Global VPL values are compared with one and two outliers observation bias, which demonstrates the efficiency and validity of proposed algorithm.

7. Conclusions

To keep up with the pace of GNSS development and GNC microsystem, we manage to participate in processing with multi-constellation GNSS and multi possible outliers in GPS/BDS/Galileo. In this paper, we propose a fast satellite selection algorithm under multi-constellation on account of Newton efficient equivalence solution for GDOP fast computation. An effective closed-form formula is proposed for GDOP approximation and fast calculation, avoiding conventional matrix inversion with large computational complexity. We take fast satellite selection algorithm and RAIM/FDE into consideration for GNSS integrity enhancement, including global and local test for the rejection of possible outliers. Global test is carried out initially and the normal solution is achieved when the residual vector does not exceed the predefined threshold. In other case, we will put local test into effect in order to detect and exclude the possible blunders. The combined methods have been used as a backbone for reliable and robust positioning results after RAIM/FDE in premise of fast satellite selection.

In summary, a novel methodology has been developed for FDE in GNC microsystem. This approach is established on the direct evaluation of subsets precheck and residual vector adjustment to conduct RAIM. Fast satellite selection algorithm is compared with traditional optimal and quasi-optimal algorithm. Static and dynamic motion trajectory is contrasted with PR bias observation. Global VPL values are

compared with one and two outliers, which demonstrates the efficiency and validity of proposed algorithm. While concerning more complex motion, we could involve adaptive motion estimation model with color noise estimation by Kalman filter instead of least squares, which could enhance the receiver performance, demanding further research.

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